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THE DISPERSION OF MATTER IN NEUTRAL AND STABLY STRATIFIED ATMOSPHERIC SURFACE LAYERS

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Abstract—The dispersion of neutrally buoyant matter in neutral and stably stratified atmospheric surface layers is described by the transient diffusion equation with an eddy diffusivity approximation. The concentration moment method, which is well suited to computer techniques, is used to solve the mass conservation equation for dispersion near the instantaneous line source at ground and elevated levels. The moment equations are solved by numerical methods for the zeroth, first, second, third and fourth moment of the longitudinal concentration distribution.

The present results agree with analytic solutions available for neutral flows. The statistical properties of the dispersing cloud are used to find the constants that appear in the Lagrangian similarity hypothesis of Batchelor for wide range of stability conditions.

NOMENCLATURE

a_1, defined for equation (20);

b, Batchelor's universal constant that appears in equation (15);

b_1, defined for equation (20);

c, Batchelor's universal constant that appears in equation (16);

c_1, defined for equation (20);

C, concentration of dispersant at a point;

C_p, p'th moment of longitudinal concentration distribution taken about $\xi = 0$;

D_n, vertical eddy diffusivity averaged over the cross-section of the flow for neutral atmosphere;

D_s, vertical eddy diffusivity averaged over the cross-section of the flow for stably stratified atmosphere;

$D\eta$, space increment that is used in numerical solutions, 0·00833;

$D\tau$, time increment that is used in numerical solutions, 0·000015–

F, flatness factor for longitudinal concentration curves;

k, von Kármán turbulence coefficient in a logarithmic velocity distribution, 0·42;

L, Monin–Obukhov stability length, equation (9);

m, exponent in power law variation of concentration;

Ri, local gradient Richardson number defined as $u_\tau^3/k(g/T)(-9/c_p\rho)$

S, skew coefficient for longitudinal concentration curves;

S, non-dimensional shear, i.e. $kz/u_T\,(du/dz)$;

t, dispersion time;

$\bar{U}$, mean velocity;

u_τ, friction velocity, square root of wall shear stress divided by fluid density;

x, longitudinal spacial coordinate;

$\bar{X}$, first moment of longitudinal concentration distribution taken about the mean of the cloud; $\bar{X} = z_n\xi + \bar{U}_t$

Y, transverse spacial coordinate;

z, vertical spatial coordinate;

$\bar{z}$, vertical mean displacement of the cloud;

$\bar{z}_0$, vertical initial mean displacement of the cloud;

z_n, reference height in neutral atmosphere;

z_s, reference height in stably stratified

atmosphere;
z_0, friction height;
α, non-dimensional stability parameter, equation (9);
β, empirical constant in log-linear law for velocity;
δ, Dirac delta function;
ζ, dimensionless transverse space coordinate, defined as y/z_n;
η_n, dimensionless vertical space coordinate, defined as z/z_n;
η_s, dimensionless vertical space coordinate, defined as z/z_s;
μ_n, dimensionless mean velocity of flow, defined as $\bar{U}z_n/D_n$;
μ_s, dimensionless mean velocity of flow, defined as $\bar{U}z_s/D_s$;
$(\mu\chi)_n$, dimensionless local velocity of flow relative to origin, $\xi = 0$, for coordinate system moving at mean dimensionless velocity μ_n;
$(\mu\chi)_s$, dimensionless local velocity of flow relative to origin, $\xi = 0$, for coordinate system moving at mean dimensionless velocity μ_s;
ξ_n, dimensionless longitudinal distance coordinate defined as $(x - \bar{U}t)/z_n$;
ξ_s, dimensionless longitudinal distance coordinate defined as $(x - \bar{U}t)/z_s$;
$\bar{\xi}$, dimensionless mean longitudinal displacement of dispersant from $\xi = 0$;
σ_x^2, variance of longitudinal concentration distributions;
σ_ξ^2, dimensionless variance of longitudinal concentration distributions;
τ_n, dimensionless dispersion time defined as $D_n t/z_n^2$ which is $ku_t t/2z_n$;
τ_s, dimensionless dispersion time defined as $D_s t/z_s^2$ which is $ku_t t/2z_s$;
Φ, function defined by equation (9);
ϕ, universal dimensionless function involving L defined by equation (17);
χ, weighting function for the local velocity, defined as $(U - \bar{U})/\bar{U}$;
ψ_n, weighting function for the local eddy diffusivity, defined as ε_{v_n}/D_n;
ψ_s, weighting function for the local eddy diffusivity, defined as ε_{v_s}/D_s.

Subscripts
n, neutral atmospheric conditions;
p, order of the moment of longitudinal concentration distribution taken about $\xi = 0$; integers;
s, stably stratified atmospheric conditions.

Superscript
$\bar{\ }$ cross-sectionally averaged.

INTRODUCTION

In the past decade considerable progress has been made in the prediction of dispersion of matter in turbulent shear flows. The practical applications of this literature to pollution control studies in natural streams and in the atmosphere are becoming more numerous. The means of attacking the problem has usually involved seeking solutions, for specific initial and boundary conditions, of the Eulerian transient diffusion equation. These analysis as applied to neutral atmospheric conditions, to fully developed channel and pipe flows have usually been based on eddy diffusivity approximation using Reynolds' analogy and semi-empirical solutions for the mean flow field.

The difficulty of obtaining solutions to the transient diffusion equation has encouraged use of concentration moment transformations which convert the equation to a more tractable system. Aris [1] was the first to use the moment transformations in laminar pipe flows. Sayre [2, 3] employed Aris' moment transformations to describe the behavior of the dispersant in a turbulent, open channel flow. Atesmen [4, 5] extended G. I. Taylor's [6] analysis in turbulent pipe flows in much the same manner as Sayre enlarged on Elder's analysis of open channel flow [7].

Recently Chatwin [8] applied the moment technique to puff dispersion in an idealized neutral atmosphere. He obtained analytic solutions to the zeroth, first, and second moments for the marginal distribution of the dispersing cloud that is released from an instantaneous point source in the constant stress region. He also showed that many of the results of the Lagrangian similarity theory of Batchelor [9, 10] can be derived without the dimensional arguments used by previous investigators. Chaudry [11] obtained analytical solutions to the zeroth and the first moments of the cloud in a stably stratified atmosphere.

In this paper, the zeroth, first, second, third and fourth moment of a dispersing cloud are obtained by using an eddy diffusivity approximation, Reynolds' analogy, and semiempirical solutions for the mean flow field. The mean flow field for the neutral atmospheric surface layer satisfies the logarithmic wind profile and the resulting eddy diffusivity profile for the constant stress region. The mean flow field for the stably stratified atmosphere surface layer is assumed to follow the log-linear profile of Monin and Obukhov [12] and the resulting eddy diffusivity profile for a constant stress region. Local and cross-sectionally averaged statistical properties are reported and discussed. The "universal" constants that appear in Lagrangian similarity hypothesis are also investigated.

PRESENT ANALYSIS

One of the most direct theoretical approaches to the dispersion process is based on the principle of conservation of mass. The formulation of the turbulent dispersion equation into its correspondent concentration moment equation for the marginal longitudinal concentration distributions is detailed in several references, e.g. [1, 2, 4, 8, 11]. The concentration moment formulation requires that the flow field be axially homogeneous but the eddy diffusivity may vary in the transverse directions. Thus in this work, ψ will be a specified function of η which satisfies the constant shear stress model and Reynolds analogy. It is worth noting that the eddy diffusivity in the transverse direction in equation (1) could in principle also be time dependent as required by Taylor's continuous movement theory of diffusion, e.g. [13], but there does not appear to be sufficient atmospheric data on Lagrangian spectral density functions to warrant this added complexity in the present study.

$$\frac{\partial C_p}{\partial \tau} = \frac{\partial}{\partial \eta}\left(\psi \frac{\partial C_p}{\partial \eta}\right) + p\mu\chi C_{p-1} + p(p-1)\psi C_{p-2} \qquad (1)$$

where C_p is the pth integer moment of the marginal concentration distribution with respect to ξ, namely,

$$C_p(\eta,\tau) \equiv \int_{-\infty}^{+\infty} \xi^p C(\xi,\eta,\tau)\,\mathrm{d}\xi. \qquad (2)$$

Thus $C_0(\eta,\tau)$ represents the transient mass distribution in the vertical direction along the axial centerline; $C_1(\eta,\tau)$ is the mean position of the cloud as a function of vertical position and dispersion time. Note that the lateral diffusion must be accounted for before the transient point source concentration field, $C(\xi,\eta,\zeta,\tau)$, can be estimated from the integer moments of the axial marginal concentration distribution $C_p(\eta,\tau)$. In other words,

$$C(\xi,\eta,\tau) = \int_{-\infty}^{+\infty} C(\xi,\eta,\zeta,\tau)\,\mathrm{d}\zeta. \qquad (3)$$

The procedure followed here is similar to that used by Chatwin [8] for the neutral flow boundary layer.

The initial conditions for the point source are

$$\begin{aligned} C_p(\eta,0) &= \delta(\eta-\eta_0) \qquad \text{for } p = 0 \\ C_p(\eta,0) &= 0 \qquad \text{for } p > 0 \end{aligned} \qquad (4)$$

where $\delta(\eta-\eta_0)$ is the Dirac delta function and η_0 is the location of the point source. The boundary conditions are

$$\frac{\partial C_p}{\partial \eta} = 0 \qquad \text{at} \quad \eta = 0 \quad \text{and at} \quad \eta = 1. \qquad (5)$$

The boundary condition at $\eta = 1$ holds in case of an inversion. The solutions for the neutral atmosphere are essentially correct before this boundary condition starts to effect the dispersing cloud (e.g. Figs. 4b and 17), by reflecting mass at the upper boundary.

The concentration moment solutions for $p = 0, 1, 2, 3$ and 4 of equation (1) in a neutral and in a stably stratified atmosphere have been obtained numerically by using the momentum transport properties outlined below.

Throughout this study the turbulent Schmidt number has been assumed to be unity. This assumption has been criticized widely. However, recent studies by Webb [14] show that the ratio of the eddy diffusivity for heat to the eddy diffusivity for momentum remains constant, essentially equal to unity, over a wide range of thermal stratification.

The mean flow field for the neutral atmospheric surface layers is given by

$$\mu\chi = \frac{2}{k^2}(\ln\eta + 1) \tag{6}$$

where k is von Kármán's constant. The resulting eddy diffusivity profile for the constant stress region is given by

$$\psi = 2\eta. \tag{7}$$

The mean flow field for the stably stratified atmospheric surface layers comes from the well-known Eulerian similarity theory which was developed by Monin and Obukhov [12].

$$\mu\chi = \frac{2}{k^2\Phi}[\ln\eta + 1 + \alpha(\eta - 0{\cdot}5)] \tag{8}$$

where α is the stability parameter, namely

$$\alpha = \beta z_s/L$$

and

$$\Phi = [\alpha - \ln(1 + \alpha)]/(\alpha^2/2). \tag{9}$$

The resulting eddy diffusivity profile for the constant stress region of the stably stratified flow field is given by

$$\psi = \frac{2\eta}{\Phi(1 + \alpha\eta)}. \tag{10}$$

It has been found by Webb [14] that the log-linear wind profile of Monin and Obukhov, equation (8), is valid for z/L values between $-0{\cdot}03$ and $+1$ which includes a wide range of stable conditions, and the Monin–Obukhov coefficient β has a value of 5·2 in stable conditions. Therefore, the α values in this study range from zero, neutral conditions, to seven, very stable conditions.

BEHAVIOR OF THE STATISTICAL PROPERTIES OF A DISPERSING CLOUD FROM AN INSTANTANEOUS POINT SOURCE AT GROUND LEVEL

All the concentration moments are calculated in a frame of reference which is moving with the mean velocity; note the definition of ξ in the nomenclature list. The results can easily be converted to a frame of reference which is moving with the mean displacement of the cloud as follows.

$$\frac{C_1}{C_0} = (\bar{X} - \bar{U}t)/z_s \tag{11}$$

and

$$\frac{C_2}{C_0} = \frac{\sigma_x^2}{z_s^2} + \left(\frac{C_1}{C_0}\right)^2, \text{ etc.}$$

This latter procedure was used by Chatwin [8] and Csanady [15] for atmospheric dispersion, while most investigations of dispersion in pipes and channels [3, 5] have adopted the frame of reference used herein.

It is very difficult to compare the statistical properties of a cloud in a neutral atmosphere with one in a stably stratified atmosphere because the Monin–Obukhov length scale L goes to infinity for the neutral case. Furthermore, it is hard to devise a momentum similarity between the different stability conditions. In boundary-layer type flows, the free stream velocity, and in confined flows such as in pipes and in channels, the mean velocity have been the customary momentum parameter. However, none of these velocities are customarily used for the atmosphere, but rather all attention is traditionally given to the local velocity values.

In this study, z_n is chosen as an arbitrary height for the neutral atmosphere which is typically between 50–500 ft. z_s is chosen as an arbitrary height for the stably stratified atmosphere which is typically equal to L. As the stratification increases, L decreases; that is, the flow is confined more and more closely to the ground. With these arbitrary upper heights, cross-sectionally averaged mean velocities can be defined as

$$\overline{U}_n = \frac{1}{z_n}\int_0^{z_n} U\,\mathrm{d}z \qquad \text{for neutral flows, and}$$

$$\overline{U}_s = \frac{1}{z_s}\int_0^{z_s} U\,\mathrm{d}z \qquad \text{for stably stratified flows.}$$

The mean velocity for the neutral case $\overline{U}_n$ is taken to be equal to the mean velocity for the stable case $\overline{U}_s$ in order to be able to compare the results; that is,

$$z_n/z_s = \exp(0{\cdot}5\,\alpha) \qquad (12)$$

by assuming $u_{\tau_n} \approx u_{\tau_s}$, $z_0/z_n \cong 0$, and $z_0/z_s \cong 0$. Also most of the plots are shown in the coordinate system of the neutral atmosphere, namely $\eta_n = z/z_n$ and $\tau_n = D_n t/z_n^2$ for easier comparison

$$\frac{\tau_s}{\tau_n} = \Phi \exp(0{\cdot}5\,\alpha). \qquad (13)$$

The comparison of statistical properties in stratified flows can be made in several different ways, namely by setting $U_s(z_n) = U_n(z_n)$ or $U_s(z_s) = U_n(z_n)$, etc. These relations change only equations (12) and (13). Therefore, the reader can convert the statistical properties in a reference frame of his wish.

If the mean temperature and the mean velocity profiles are assumed to remain similar throughout the range of thermal stability that is of interest here, the Richardson number Ri profiles become

$$Ri = z/L/(1 + \beta z/L). \qquad (14)$$

The Ri profiles are shown in Fig. 1. As α increases, $L\,Ri/z_s$ decreases so Ri increases; that is, $\beta Ri/\alpha = L\,Ri/z_s$.

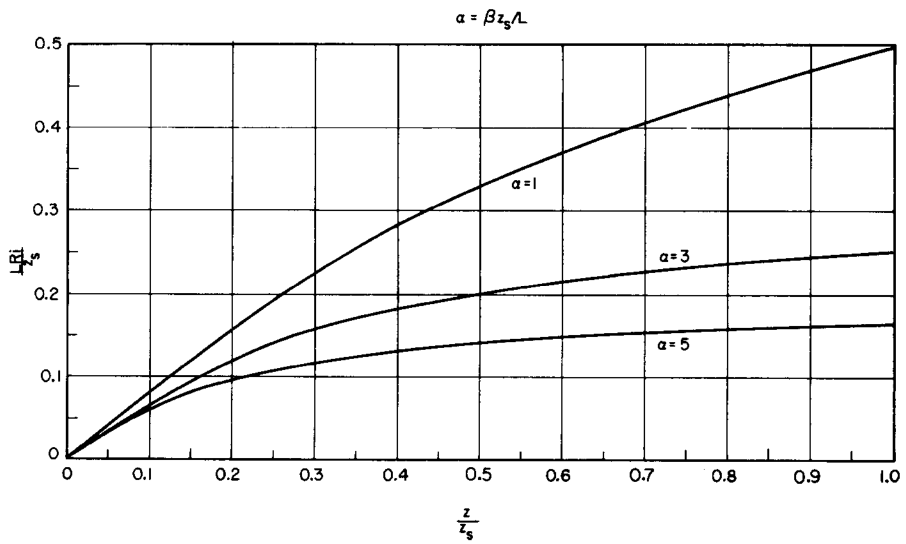

FIG. 1. Richardson number as a function of z/z_s for different stability conditions.

In all runs the strength of the instantaneous point source was chosen such that the cross-sectionally averaged zeroth moment always remained equal to unity. The normalization of $C_0(\eta, \tau)$ averaged over the vertical coordinate $x = \bar{U}t$, for ground sources under various stability conditions is shown in Fig. 3 in terms of a power index. These power indices are valid only for the given range of τ_n.

In Fig. 4a, the vertical distributions of the

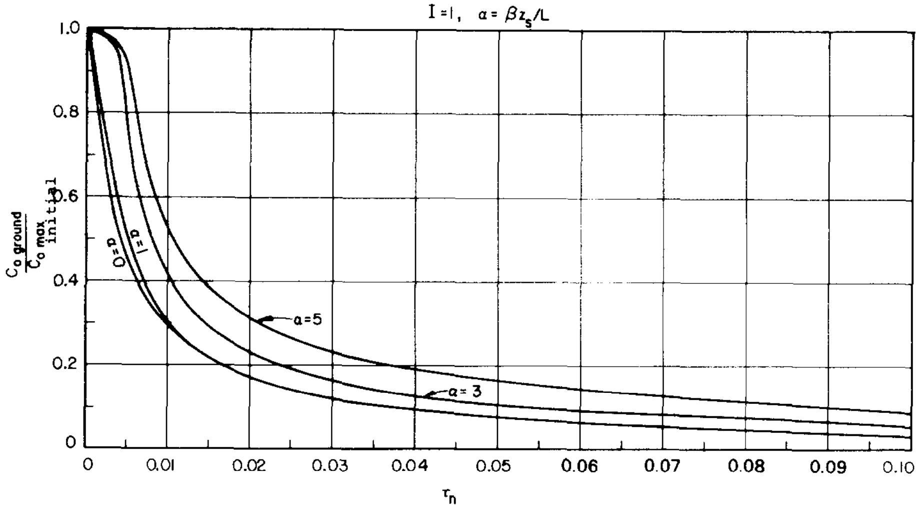

FIG. 2. Comparison of the amount of dispersant that stays on the ground for different stability conditions.

corresponds to the release from an infinite line source parallel to the y axis. To obtain absolute values of concentration downstream from a point source of release at the origin, it would be necessary to multiply $C_0(\eta, \tau)$ by the fraction of the injected mass which has diffused off the semi-infinite infinitesimal slab of thickness dy and height z_n (or z_s) which extends along the axial coordinate from $x = -\infty$ to $x = +\infty$. In other words, to obtain absolute values of concentration in the point source configuration we must recall that $C_p\ (\eta, \tau)$ are axial moments of a marginal concentration distribution as defined by equations (2) and (3).

In Fig. 2, the amount of dispersant that stays on the ground is shown as a function of τ_n for different stabilities. As stability and therefore α increases, $C_{0\,\text{ground}}$ increases as expected.

The variation of maximum concentration with time or with axial distance downwind, i.e.

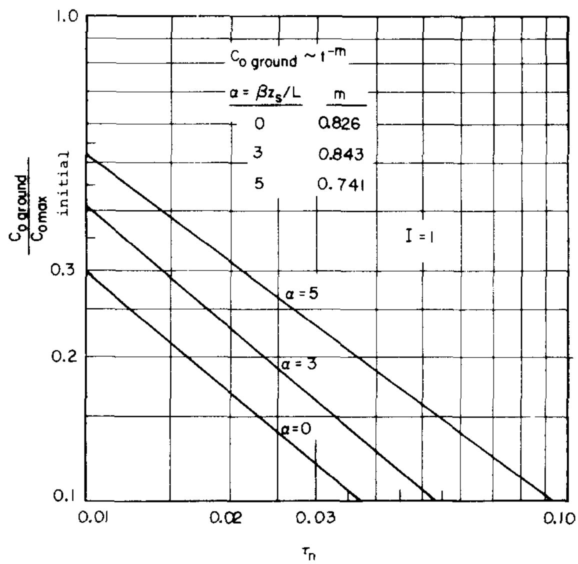

FIG. 3. Comparison of the amount of dispersant that stays on the ground as a function of dispersion time.

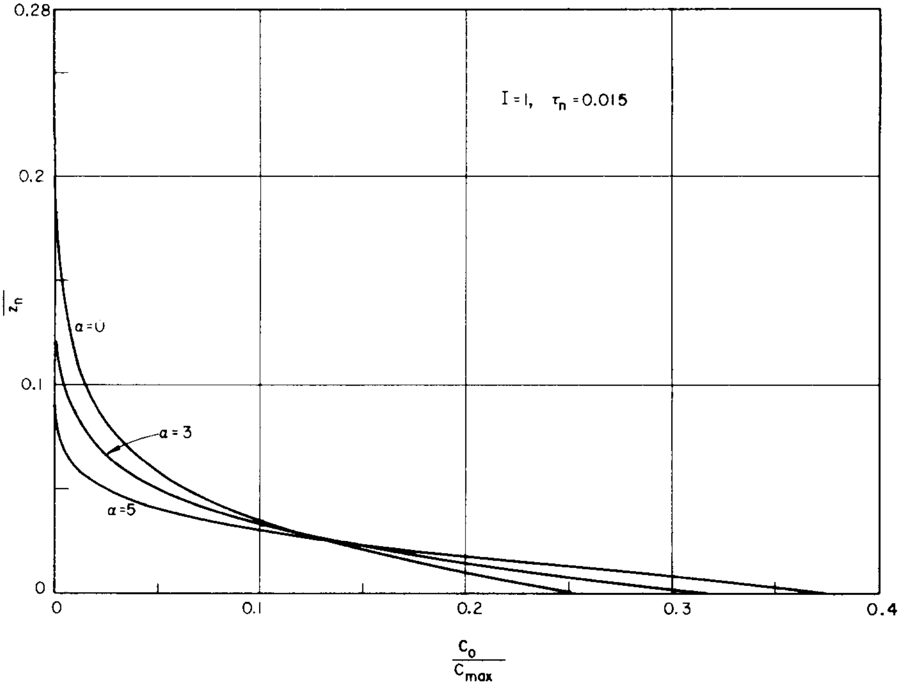

FIG. 4a. Vertical distributions of the amount of dispersant at $\tau_n = 0{\cdot}015$ for different stability conditions.

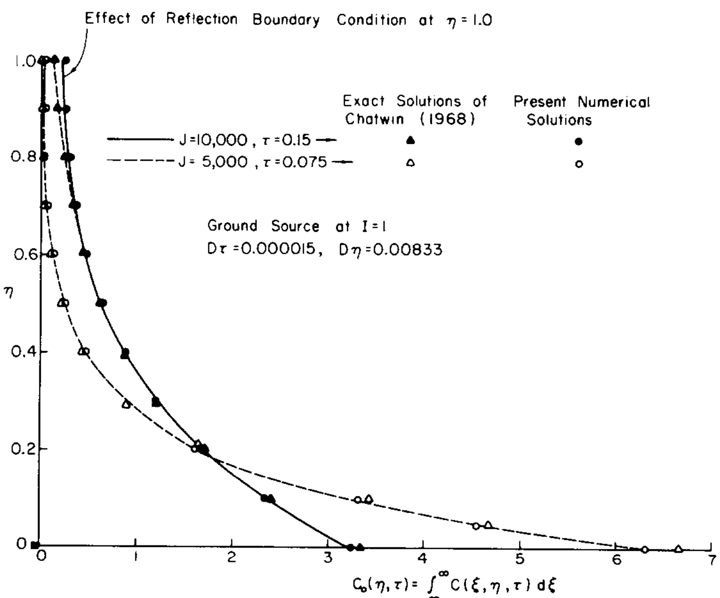

FIG. 4b. Comparison of zeroth moments between Chatwin's (1968) exact solutions and present numerical solutions for an instantaneous ground point source in a neutral atmosphere.

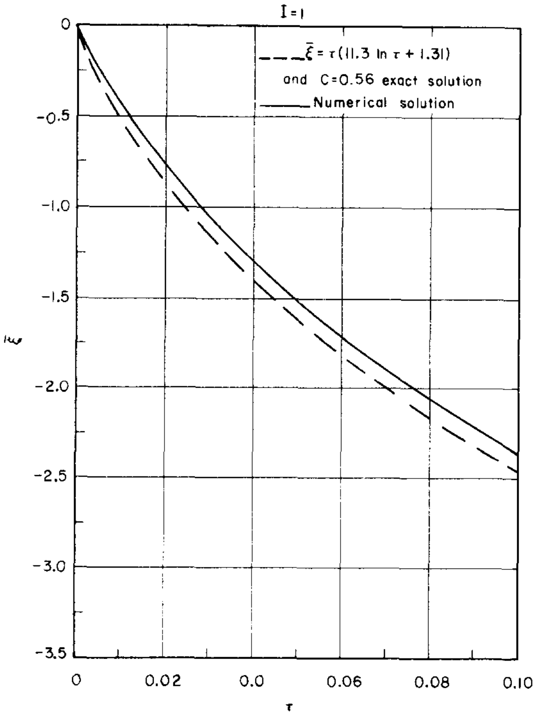

FIG. 5. Comparison of numerical and analytical cross-sectionally averaged first moments as a function of dispersion time.

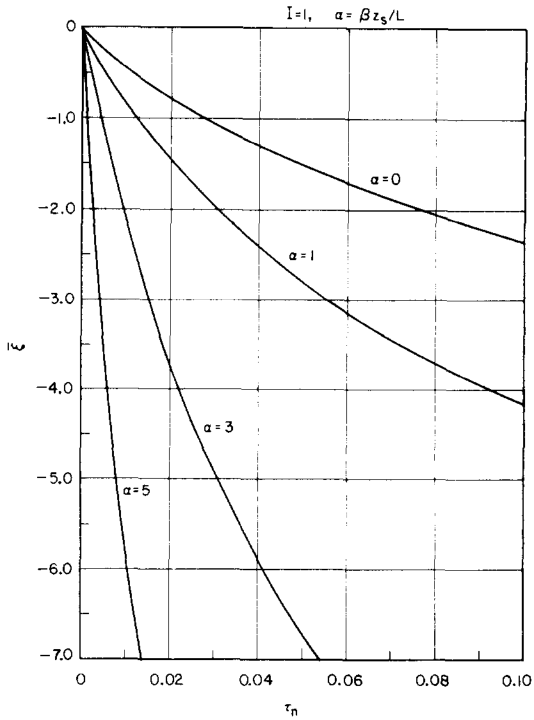

FIG. 6. Comparison of cross-sectionally averaged first moments for different stability conditions as a function of dispersion time.

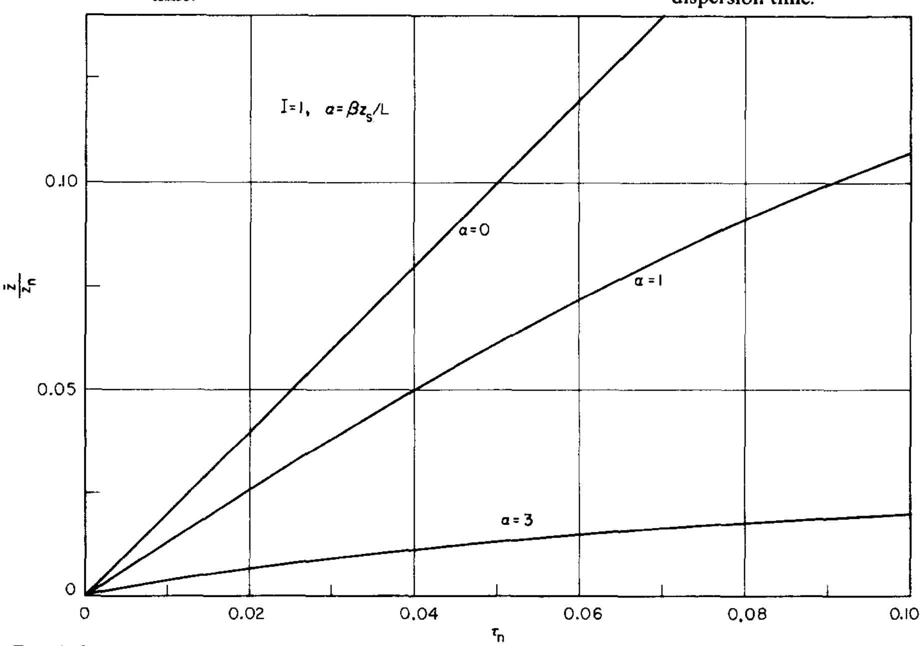

FIG. 7. Comparison of cross-sectionally averaged vertical mean displacements for different stability conditions as a function of dispersion time.

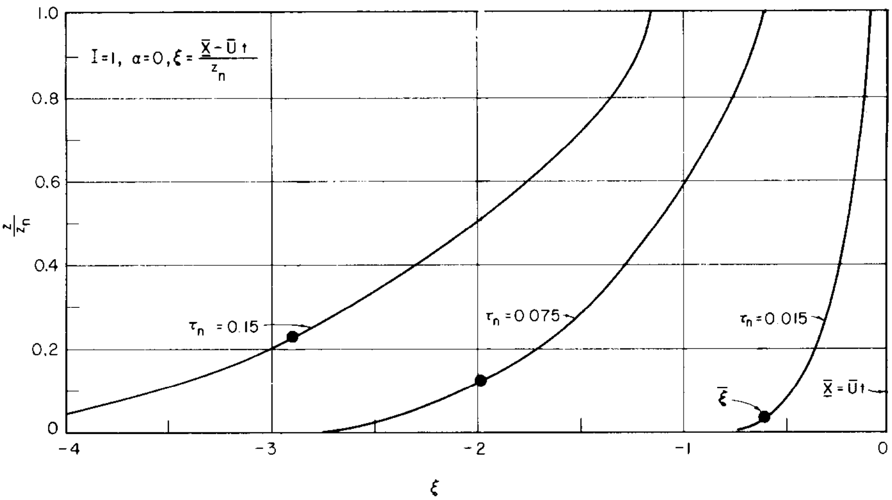

FIG. 8. Vertical distributions of the first moments at different dispersion times in a neutral atmosphere for ground level sources; cross-sectional averaged mean displacements $\bar{\xi}$ are shown as closed circles.

zeroth moments are compared for different stability conditions at $\tau_n = 0{\cdot}015$. Physically, these curves correspond to the relative vertical distribution of mass downstream from a ground level source at a specified non-dimensional time. More dispersant remains closer to the ground with increasing stability or α.

The behavior of the zeroth moments in a neutral atmosphere are also compared in Fig. 4b with Chatwin's [8] analytical solution where he used the upper boundary condition $C \to 0$ as $z \to \infty$. The present numerical solutions agree very closely with the analytical solution. However, the use of a reflection boundary

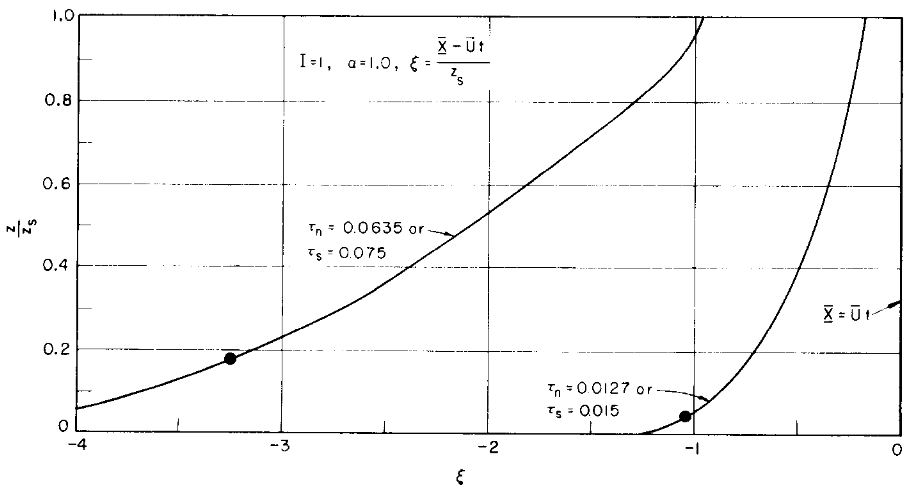

FIG. 9. Vertical distributions of the first moments at different dispersion times in a stably stratified atmosphere; cross-sectional averaged mean displacements $\bar{\xi}$ are shown as closed circles.

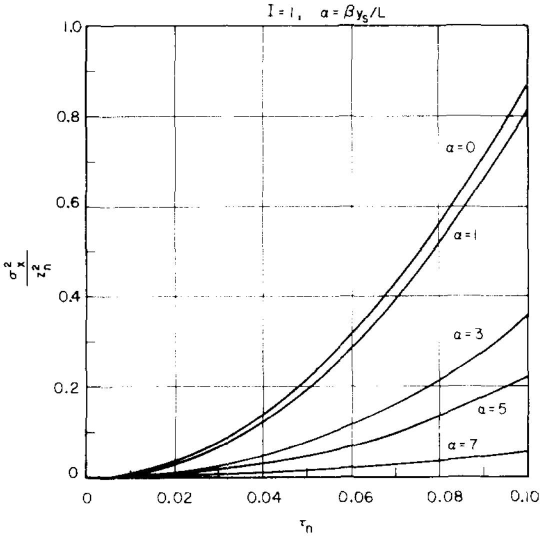

FIG. 10. Cross-sectionally averaged variance of longitudinal concentration distribution as a function of dispersion time.

condition in the present numerical solutions of equation (1) at a reference height z_n causes the numerical solutions to start to deviate appreciably for $\tau_n \geqslant 0{\cdot}15$ from Chatwin's exact solutions. Therefore, all the statistical properties that are reported in this study are for $0 \leqslant \tau_n \leqslant 0{\cdot}15$.

The cross-sectionally averaged mean displacement of the cloud from $x = \bar{U}t$, i.e. from the observer riding the mean velocity, is shown in Fig. 5. The displacement of the cloud is slower than the displacement of the observer. The numerical solution is compared with the analytical solution assuming $b = k$ and $c = 0{\cdot}56$. The discrepancy between the two curves is due to the values of c which decreases with time and approaches the long time asymptotic value of 0·56. The behavior of the Lagrangian similarity constants will be discussed later in more detail. The cross-sectionally averaged longitudinal mean displacement of the cloud from the observer is shown in Fig. 6 for various stability conditions. As α increases, the cloud does not diffuse as rapidly as it would in a neutral atmosphere. The cloud then moves with a mean velocity which is weighted to the slow speeds near the ground and deviates appreciably from the vertically averaged mean wind.

In Fig. 7, the cross-sectionally averaged vertical mean displacement of the cloud $\bar{z}$ is shown. A comparison of vertical mean displacements for the same mean velocity under neutral and stable conditions reveals that the vertical spread is greatly reduced due to the effect of thermal stratification on cloud growth. Figures

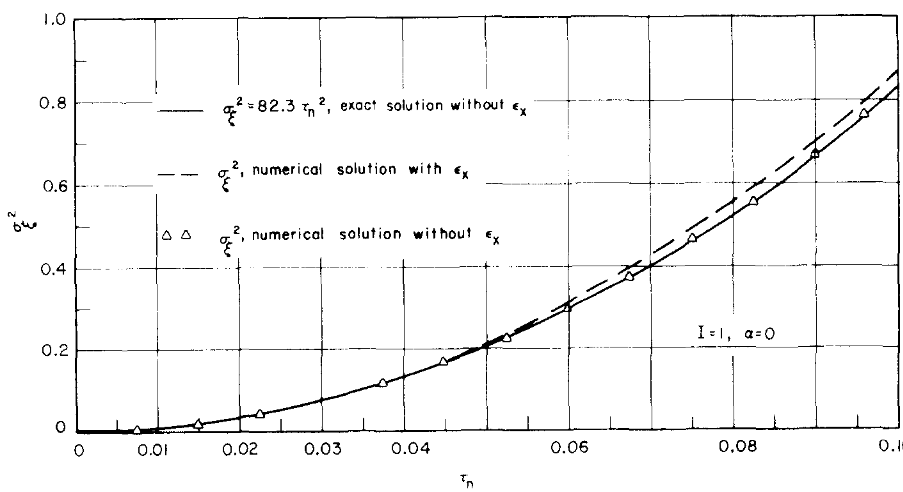

FIG. 11. Comparison of numerical and analytical second moments as a function of dispersion time.

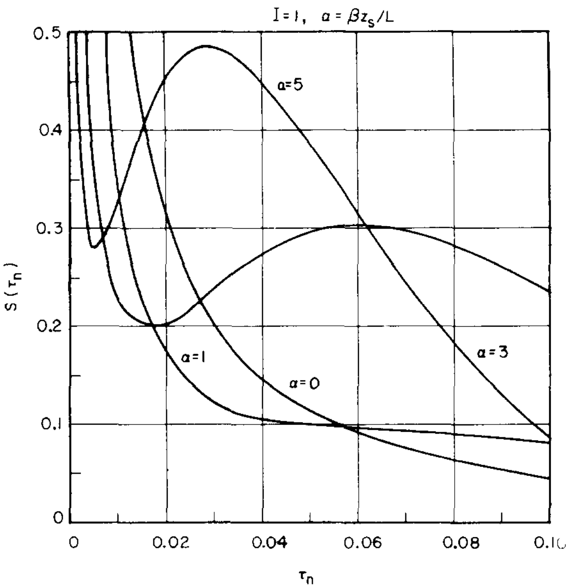

FIG. 12. Skew coefficient of longitudinal concentration distributions as a function of dispersion time.

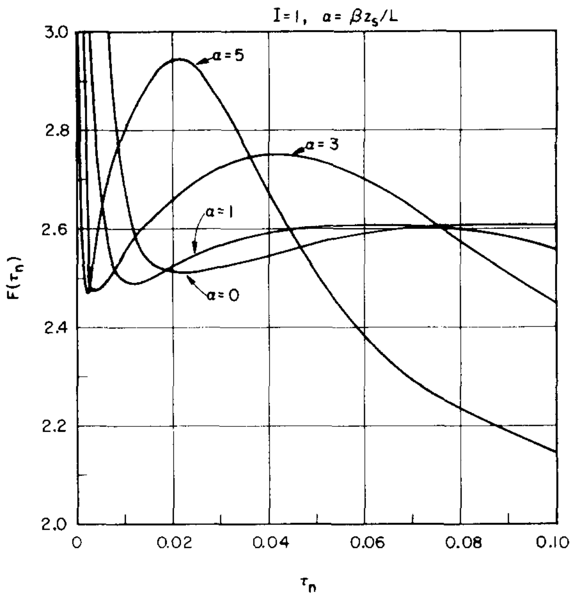

FIG. 13. Flatness coefficient of longitudinal concentration distribution as a function of dispersion time.

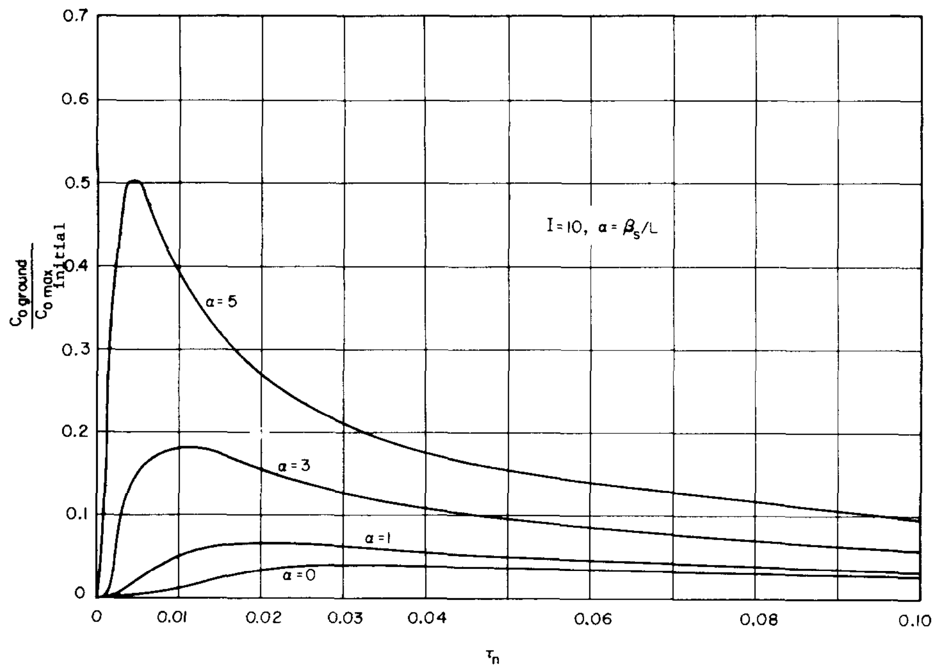

FIG. 14. Comparison of the amount of dispersant that stays on the ground from an elevated source as a function of dispersion time.

8 and 9 show the vertical distributions of longitudinal local displacements of the cloud at different times for two stability conditions, $\alpha = 0$ and $\alpha = 1$. For comparison purposes, the cross-sectionally averaged mean displacement of the cloud $\bar{\xi}$ taken from Fig. 6, are plotted as solid circles on the local vertical distributions of ξ plotted in Figs. 8 and 9.

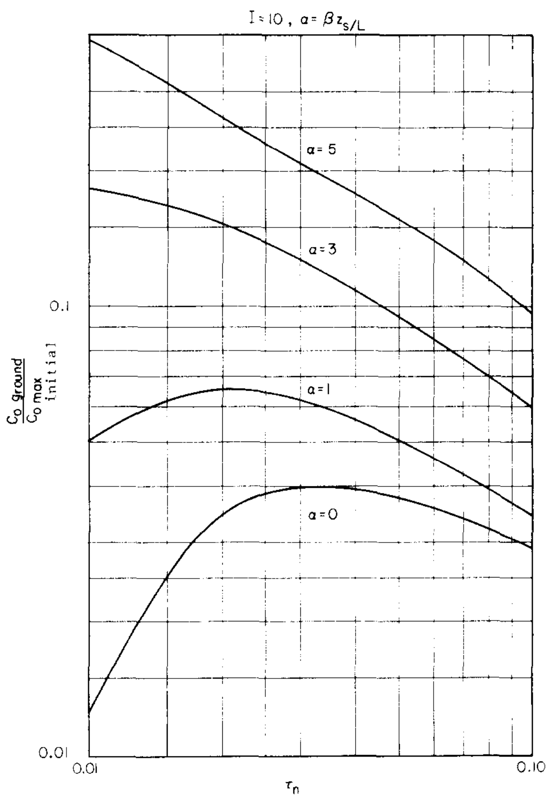

FIG. 15. Comparison of the amount of dispersant that stays on the ground from an elevated source for different stability conditions.

Clearly, the bulk of the cloud is well below the upper boundary for the cases shown.

The relative longitudinal spread of the cloud can be estimated from Fig. 10 for different stability conditions. The dispersant spreads much more slowly with increasing stable conditions. Figure 11 compares a previous analytical solution for neutral case with the numerical solution. The analytical solution was obtained by neglecting the longitudinal turbulent diffusion which was assumed to be small compared with interactive combination of advection and vertical turbulent diffusion. The addition of the longitudinal turbulent diffusion in the present numerical solution increases the variance of the cloud negligibly thereby verifying the assumption that turbulent transfer in the axial direction is small compared with what is termed by some authors as "Taylor's diffusion".

The Skew coefficient and the flatness coefficient of the vertically averaged longitudinal concentration distributions are shown in Figs. 12 and 13 respectively for several stability conditions as a function of non-dimensional dispersion time τ_n. The behavior of these higher moments show that the longitudinal concentration distributions are not Gaussian near the source. The deviations from normality can be accounted for by using the Hermite polynomial representations of the local longitudinal concentration distributions (see, Chatwin [16] and Atesmen [5]).

BEHAVIOR OF THE STATISTICAL PROPERTIES OF A DISPERSING CLOUD FROM AN INSTANTANEOUS ELEVATED POINT SOURCE

To investigate the influence of source elevation, an elevated source was located at $\eta = 0{\cdot}0833$ or the tenth vertical increment of the computational grid. Again most of the plots are shown in the coordinate system of the neutral atmosphere, namely $\eta_n = z/z_n$ and $\tau_n = D_n t/z_n^2$.

Figures 14 and 15 show the zeroth moment of the dispersant on the ground level as a function of τ_n for different stabilities. As the stability increases more and more dispersant is trapped on the ground level.

The vertical distributions of the zeroth moments are shown in Figs. 16 and 17 for different stability conditions. The neutral case is compared with Chatwin's [8] analytical solution. The two solutions agree very closely.

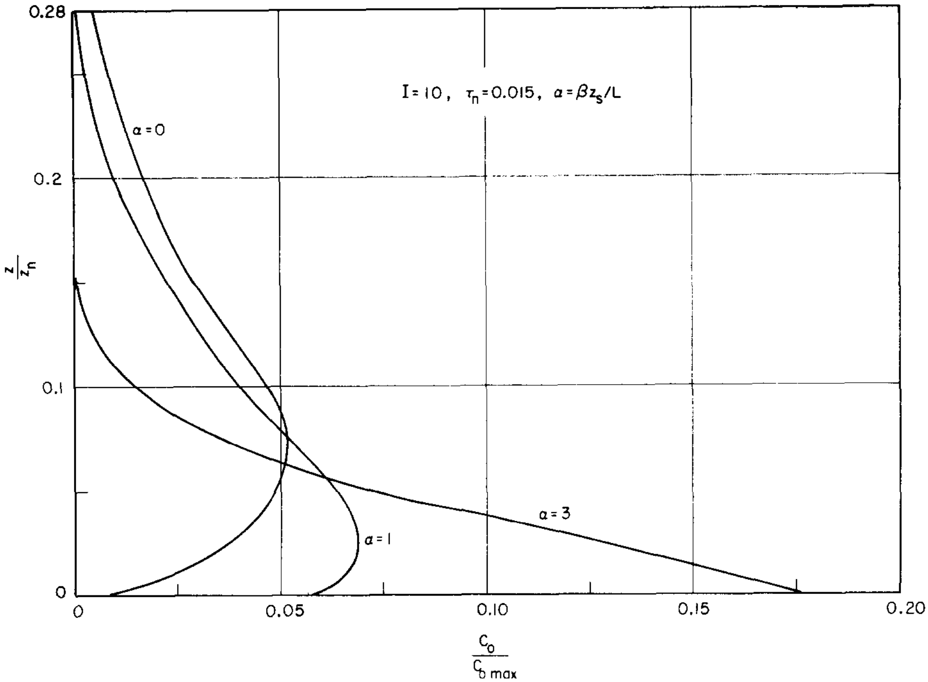

FIG. 16. Vertical distributions of the amount of dispersant at $\tau_n = 0{\cdot}015$ for different stability conditions.

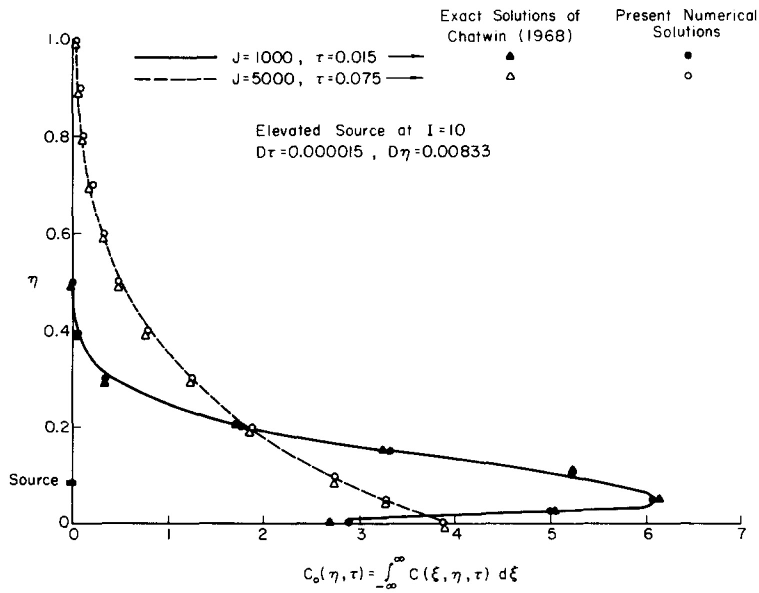

FIG. 17. Comparison of zeroth moments between Chatwin's (1968) exact solutions and present numerical solutions for an instantaneous elevations point source in a neutral atmosphere.

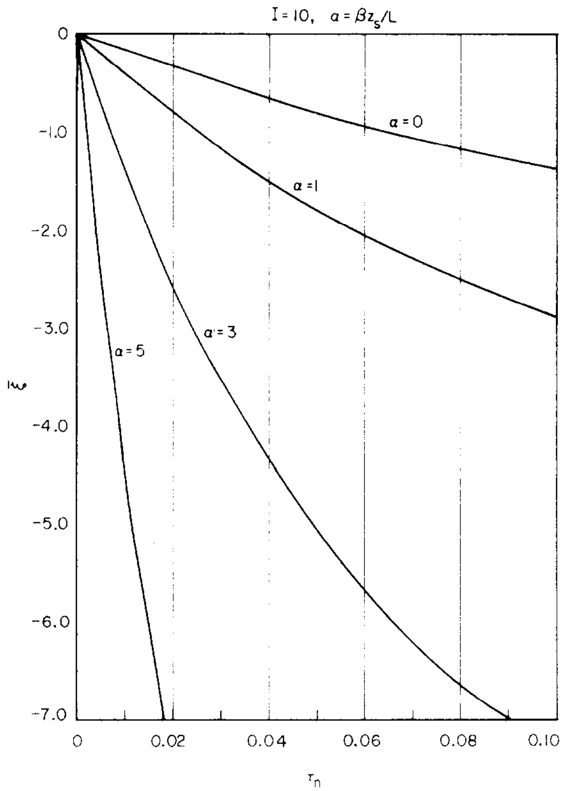

FIG. 18. Comparison of cross-sectionally averaged first moments for an elevated source as a function of dispersion time.

The cross-sectionally averaged mean displacement of the cloud from the observer is always less for an elevated source than a ground level source as shown in Fig. 18. The cloud moves downwind faster from an elevated source as expected, because the mass is injected into a region of higher velocity than a ground level source.

The cross-sectionally averaged vertical mean displacement of the cloud is shown in Fig. 19. Figures 20 and 21 show the vertical distributions of longitudinal local displacements at different times for $\alpha = 0$ and $\alpha = 1$; the cross-sectionally averaged mean displacements $\bar{\xi}$ are shown for comparison as solid circles on these distribution curves.

The variance of longitudinal concentration distribution for various stability conditions is shown in Fig. 22. The cloud spreads longitudinally relatively faster for an elevated source than a ground level source.

The skew coefficient and the flatness coefficient for the cross-sectional averaged longitudinal concentration distributions are shown in Figs. 23 and 24 respectively for several stability conditions for the elevated source.

These statistical parameters do not actually predict the longitudinal concentration distribu-

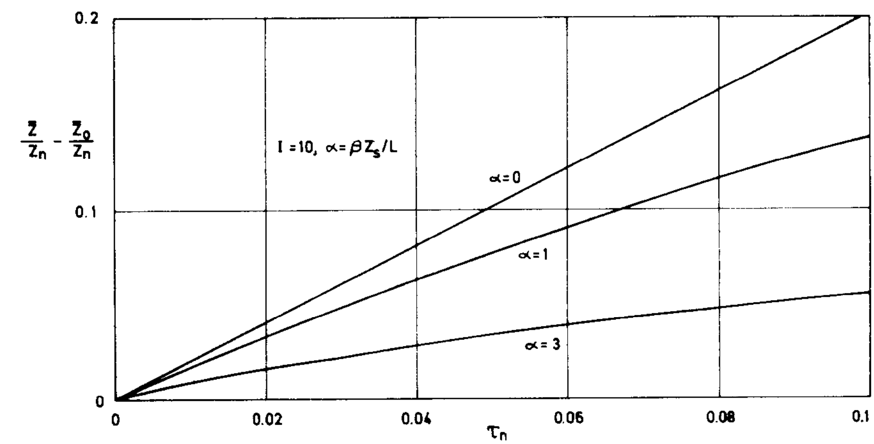

FIG. 19. Comparison of cross-sectionally averaged vertical mean displacements for an elevated source as a function of dispersion time.

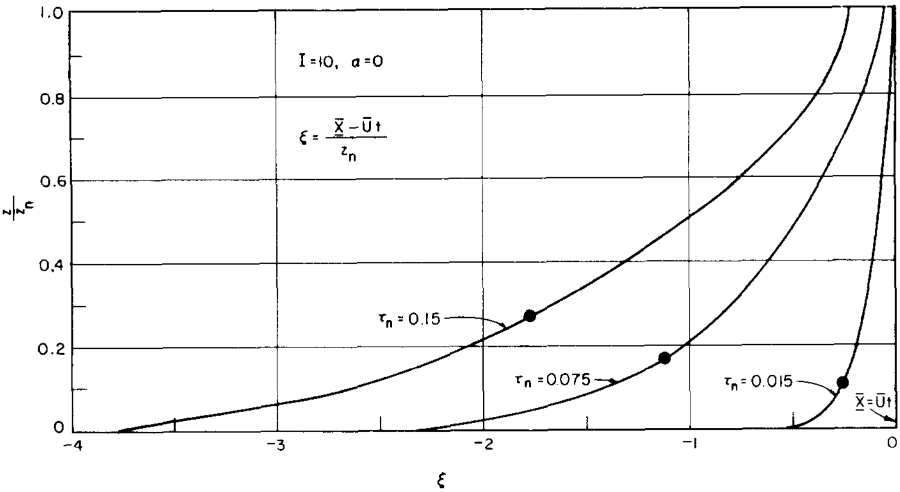

FIG. 20. Vertical distributions of the first moments at different dispersion times for an elevated source; cross-sectional averaged mean displacements $\bar{\xi}$ are shown as closed circles.

tions of the cloud, but only some measures of these curves and especially the behavior of the dispersant cloud near the injection source. However, the combination of the local moments from $C_0(\eta, \tau)$ through $C_4(\eta, \tau)$ together with the Hermite polynomial series distribution function (see Chatwin [16] and Atesmen [5]) can be used as a detailed dispersion model for neutral and for the stably stratified atmospheric surface layers. In many engineering problems dispersants are introduced intermittently or continuously into the atmosphere. Such sources cannot be approximated well by an instantaneous source, but the concentration distribu-

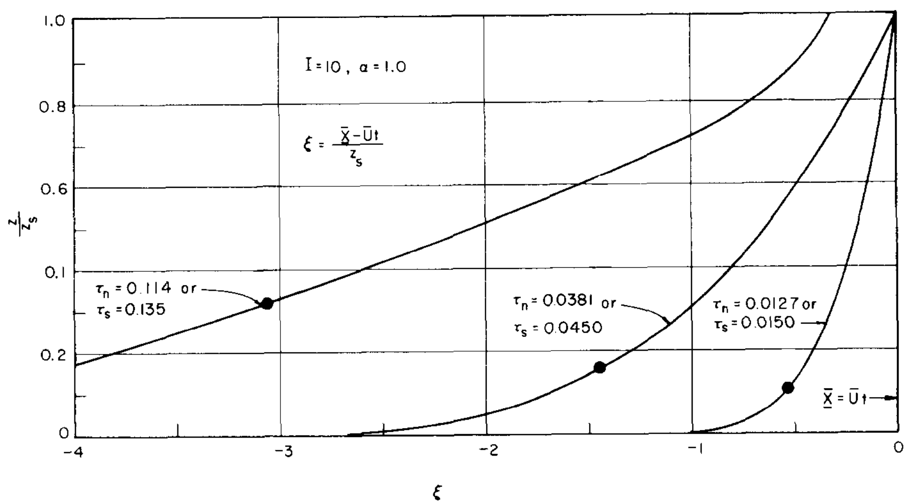

FIG. 21. Vertical distributions of the first moments at different dispersion times for an elevated source; cross-sectional averaged mean displacements $\bar{\xi}$ are shown as closed circles.

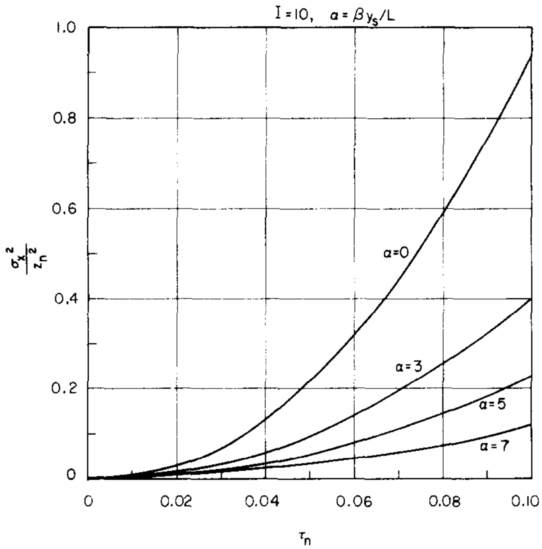

FIG. 22. Variance of longitudinal concentration distribution as a function of dispersion time for an elevated source.

tion functions resulting from instantaneous sources can be superimposed to simulate actual sources (see Batchelor [10] and Atesmen [4]).

LAGRANGIAN SIMILARITY THEORY

The Lagrangian similarity theory was suggested by Batchelor [9, 10] for a neutrally stratified atmospheric surface layer. The successful application of Eulerian similarity theory over a wide range of stability conditions, e.g. see Webb [14] and Monin [17], led to the possibility of extension of Lagrangian similarity theory to diabatic atmospheric surface layers, Gifford [18], Cermak [18] and Chaudry [11] discuss this extension in detail. Only the highlights of the theory will be reviewed here in order to compare the present numerical model to the similarity predictions.

The average vertical velocity of a particle is assumed to be uniquely determined by the shear velocity u_τ times some universal function involving L; that is

$$d\bar{z}/dt = bu_\tau\phi(\bar{z}/L) \qquad (15)$$

where $\phi(0) = 1$ for neutral conditions. Similarly,

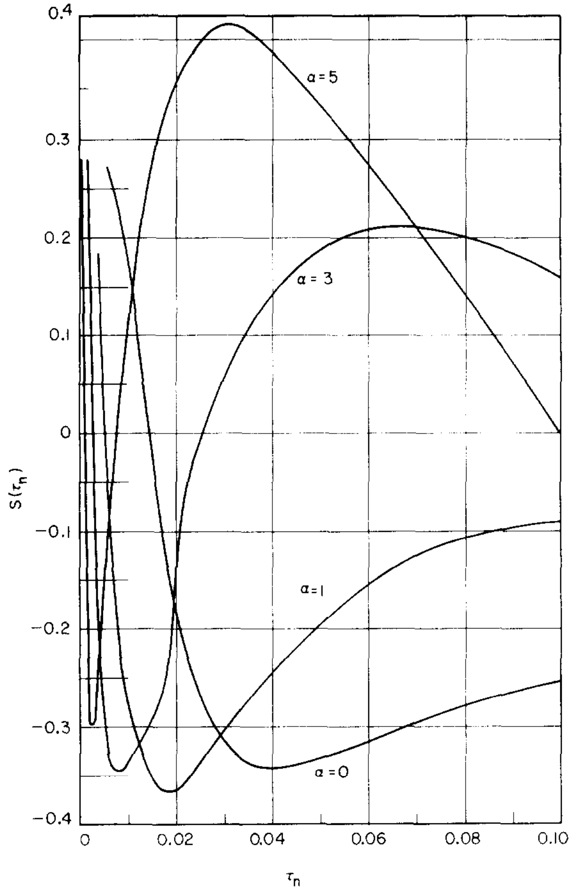

FIG. 23. Skew coefficient of longitudinal concentration distribution as a function of dispersion time for an elevated source.

the average longitudinal velocity of a particle is

$$d\bar{x}/dt = u_t/k(\ln(c\bar{z}/z_0) - \beta/L(\bar{z} - z_0)). \qquad (16)$$

The universal function ϕ is obtained using the log–linear law, namely

$$\phi(\bar{z}/L) = 1/S(\bar{z}/L) = 1/(1 + \beta\bar{z}/L) \qquad (17)$$

where $S(\bar{z}/L)$ is the nondimensional wind shear.

Combining equations (15) and (17) and integrating with the assumption that b/k is a constant yields,

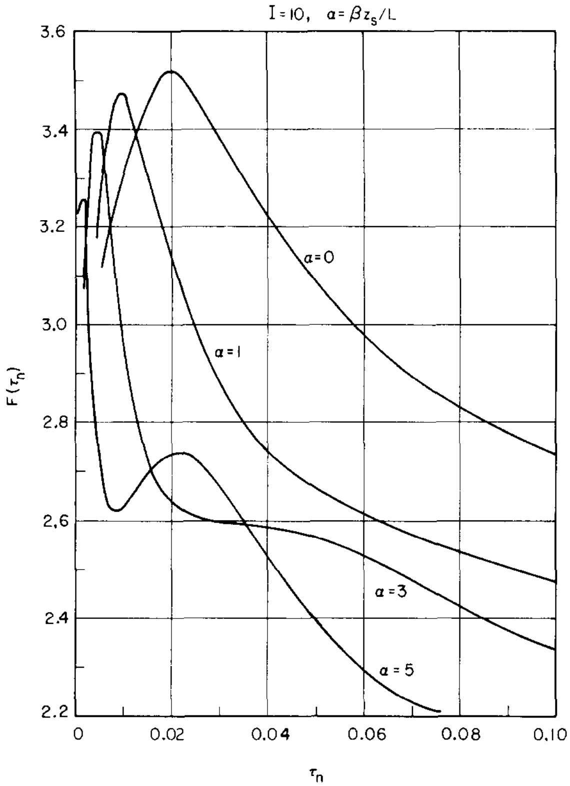

FIG. 24. Flatness coefficient of longitudinal concentration distribution as a function of dispersion time for an elevated source.

$$b/k = \left[\left(\frac{\bar{z}}{z_s} + \frac{1}{\alpha}\right)^2 - \frac{1}{\alpha^2} - \frac{2}{\alpha}\frac{\bar{z}_0}{z_s} - \frac{\bar{z}_0^2}{z_s^2}\right]\frac{\alpha\Phi}{4\tau_s} \tag{18}$$

where $\bar{z}_0$ is the initial vertical mean displacement of the cloud for an elevated source. Equation (18) simplifies to

$$b/k = \left(\frac{\bar{z}}{z_n} - \frac{\bar{z}_0}{z_n}\right)/2\tau_n \tag{19}$$

for the neutral case. Using equations (18) and (19) and the numerical moment solutions of the present work, b/k is obtained as a function of τ_n for several stability conditions. Figure 25 shows the b/k values for ground level source. Chatwin [8] found that for the neutral case, $b = k$. That is, the Eulerian diffusion equation with the model assumptions concerning velocity distributions and mass diffusivities used herein is consistent with Lagrangian similarity theory for neutral flows when one takes Batchelor's constant b to be equal to von Kármán's constant k. As the stratification increases the value of b is not constant but rather decreases. For an elevated source, Fig. 26, b is equal to k. All the

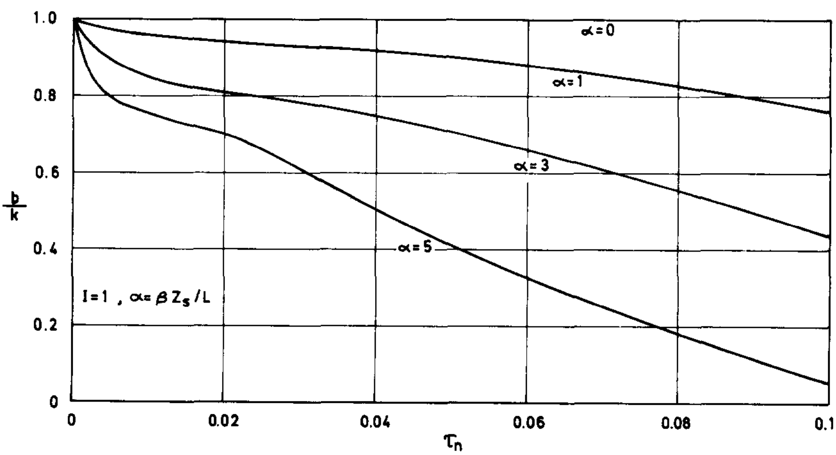

FIG. 25. Batchelor's universal constant b as a function of dispersion time for a ground source and from different stability conditions.

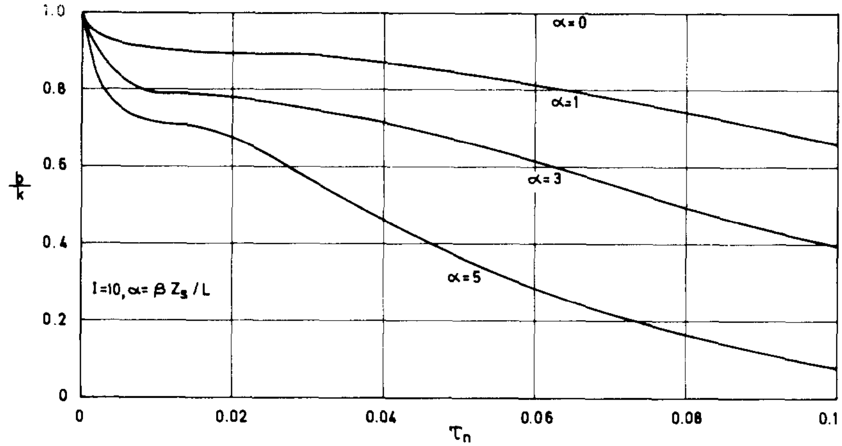

FIG. 26. Batchelor's universal constant b as a function of dispersion time for an elevated source and for different stability conditions.

b/k vs. τ_n curves appear to approach an asymptotic limit for long dispersion times. From Figs. 25 and 26, it may be concluded that Lagrangian similarity theory does not apply to short dispersion times and to stably stratified atmospheric flows.

By knowing the value of b, equation (16) can be integrated to give

$$c = \exp\left\{\frac{k^2\Phi\bar{\xi}}{2\tau_s} - \frac{2}{c_1\tau_s}\left[\frac{(a_1 + \sqrt{(b_1 + c_1\tau_s)})^2}{2}\right.\right.$$

$$[\ln(a_1 + \sqrt{(b_1 + c_1\tau_s)}) - \tfrac{1}{2}]$$

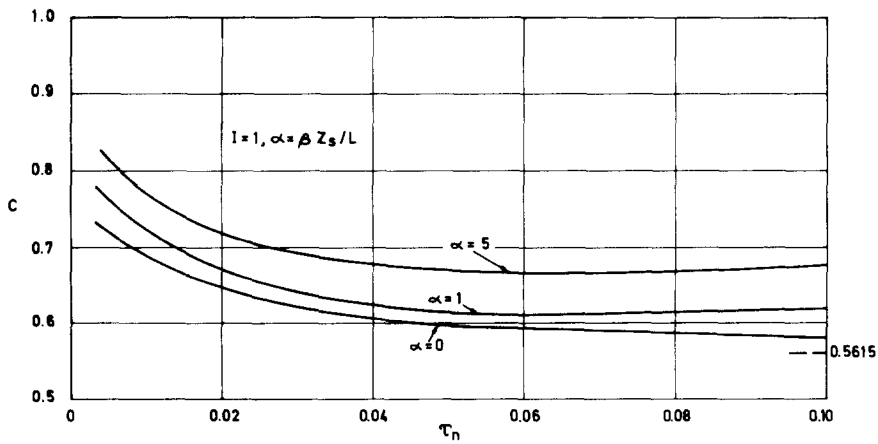

FIG. 27. Batchelor's universal constant c as a function of dispersion time for a ground source and for different stability conditions.

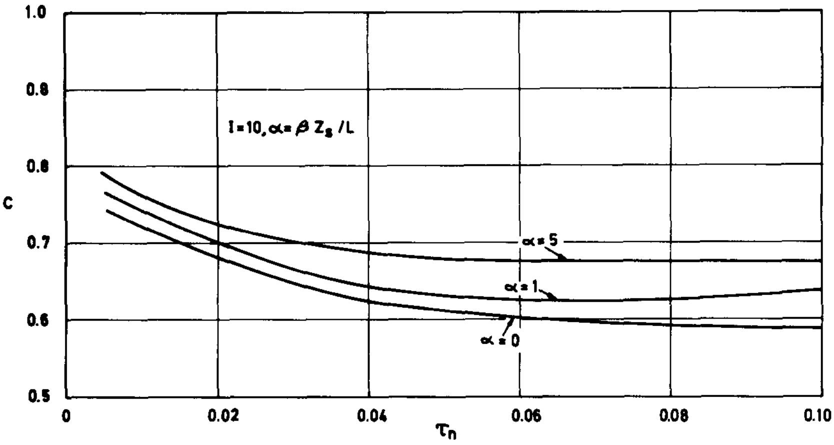

FIG. 28. Batchelor's universal constant c as a function of dispersion time for an elevated source and for different stability conditions.

$$- a_1(a_1 + \sqrt{(b_1 + c_1\tau_s)}) [\ln (a_1 + \sqrt{(b_1 + c_1\tau_s)}) - 1] - \frac{(a_1 + \sqrt{b_1})^2}{2} [\ln (a_1 + \sqrt{b_1}) - \tfrac{1}{2}] + a_1(a_1 + \sqrt{b_1}) [\ln (a_1 + \sqrt{b_1}) - 1\Big] + \frac{2}{3c_1\tau_s} [(b_1 + c_1\tau_s)^{\frac{3}{2}} - b_1^{\frac{3}{2}}] + 0{\cdot}5\alpha\Big\}, \quad (20)$$

where

$$a_1 = -1/\alpha, b_1 = \frac{1}{\alpha^2} + \frac{2}{\alpha}\frac{\bar{z}_s}{z_s} + \frac{\bar{z}_0^2}{z_s^2},$$

and

$$c_1 = \frac{4b}{\alpha k \Phi}.$$

Equation (20) simplifies to

$$c = \exp\left\{\frac{k^2\xi}{2\tau_n} - \frac{k}{2b\tau_n}\left[\left(\frac{2b}{k}\tau_n + \frac{\bar{z}_0}{z_n}\right) \ln\left(\frac{2b}{k}\tau_n + \frac{\bar{z}_0}{z_n}\right) - \left(\frac{2b}{k}\tau_n + \frac{\bar{z}_0}{z_n}\right) - \frac{\bar{z}_0}{z_n}\ln\frac{\bar{z}_0}{z_n} + \frac{\bar{z}_0}{z_n}\right] - 1\right\} \quad (21)$$

for the neutral atmosphere. Using equations (20) and (21), together with vertical and longitudinal mean displacements of the cloud obtained numerically and the b/k values previously shown, c is obtained as a function of τ_n for several stability conditions in Figs. 27 and 28. The value of c in a neutral atmosphere for a ground level source approaches an asymptotic limit of 0·56 for long dispersion times as was found by Chatwin [8]. The value of c can be described physically as the difference between the average horizontal velocity of the particles and log–linear wind velocity at the average height $\bar{z}$, namely

$$\mathrm{d}\bar{x}/\mathrm{d}t - \bar{u}(\bar{z}) = u_\tau/k \ln (c). \quad (22)$$

Both for the ground level source, Fig. 27, and for the elevated source, Fig. 28, the values of c stay the same. The c values vary with stability and with dispersion time. As the stability increases, the average horizontal velocities of the particles approach the log–linear wind velocity, i.e. equation (22). Clearly, Lagrangian similarity predictions involving constants b and c are generally not consistent with the Eulerian diffusion model used here for stably stratified flows.

CONCLUDING REMARKS

The characteristics of a cloud of neutrally buoyant dispersant have been calculated numerically using the concentration moment technique. The transient Eulerian diffusion equation with an eddy diffusivity approximation was assumed to apply to the constant stress region of an atmospheric boundary layer. The model utilizes semi-empirical wind profiles, logarithmic for neutral flows and log–linear for stably stratified flows, and a turbulent Schmidt number of unity. The technique demonstrated here could easily be applied to more complex shear flow models, if there was experimental evidence which warranted it, so long as the flow field is axially homogeneous over the region of interest.

The dispersing cloud in a stably stratified flow grows much slower than under neutral flow conditons. In all cases, the interaction of the vertical turbulent diffusion with the vertically varying axial mean flow field causes a horizontal dispersion which is many times that which would occur due to axial turbulent diffusion. The concentration distributions are complex, but using the moments of the concentration distributions presented herein, good estimates of the most probable cloud shape can be made. The clouds are highly skewed and are not well represented by a Gaussian concentration distribution; this is especially true for elevated sources in the shear region.

The "constants" of Lagrangian similarity theory are shown to be strongly dependent on the dispersion time and degree of stability. Only for the special case of neutral flows are the predictions of the present model consistent with those of Lagrangian similarity theory.

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LA DISPERSION DE MATIERE DANS DES COUCHES SUPERFICIELLES ATMOSPHERIQUES NEUTRES ET STRATIFIEES DE FACON STABLE

Résumé—La dispersion de matière librement mouvante dans des couches atmosphériques superficielles neutres et stratifiées de façon stable est décrite par l'équation de diffusion transitoire avec une approximation sur la diffusivité par turbulence. La méthode du moment de concentration qui s'adapte bien aux techniques des ordinateurs est utilisée pour résoudre l'équation de conservation de masse pour la dispersion près de la source linéaire instantannée à des niveaux près du sol et élevés. Les équations des moments sont résolues par des méthodes numériques pour les moments d'ordre zéro, un, deux, trois et quatre de la distribution longitudinale de concentration.

Les résultats s'accordent avec des solutions analytiques valables pour des écoulements neutres. Les propriétés statistiques du nuage dispersant sont utilisées pour trouver les constantes qui apparaissent dans l'hypothèse de similarité lagrangienne de Batchelor pour un large domaine de conditions de stabilité.

DIE STOFFVERTEILUNG IN NEUTRALEN UND STABILEN OBERFLÄCHENSCHICHTEN DER ATMOSPHÄRE

Zusammenfassung—Die Verteilung von Teilchen ohne Auftrieb in neutralen und stabilen Oberflächenschichten der Atmosphäre wird beschrieben mit Hilfe der Übergangs-Diffusions-Gleichung und einer Näherungslösung für den Turbulenzgrad. Die Konzentrations-Momenten-Methode, die sich gut für Rechenmaschinen eignet, wird benützt, um die Massenerhaltungsgleichung für die Verteilung nahe der augenblicklichen Linienquelle am Boden und in verschiedenen Höhen zu lösen. Die Momentengleichungen werden auf numerische Art gelöst und zwar für das nullte, erste, zweite, dritte und vierte Moment der Längskonzentrationsverteilung. Die vorliegenden Ergebnisse stimmen mit analytischen Lösungen für neutrale Strömung überein. Die statistischen Eigenschaften der sich verteilenden Wolke werden herangezogen um die Konstanten zu ermitteln, die in der Lagrangen Ähnlichkeitshypothese von Batchelor auftreten, und zwar für einen grossen Bereich der Stabilitätsbedingungen.

ДИСПЕРСИЯ ВЕЩЕСТВА В НЕЙТРАЛЬНЫХ И УСТОЙЧИВО СТРАТИФИЦИРОВАННЫХ ПОВЕРХНОСТНЫХ СЛОЯХ АТМОСФЕРЫ

Аннотация—Дисперсия нейтрально плавающего вещества в нейтральных и устойчиво стратифицированных поверхностных слоях атмосферы описывается нестационарным уравнением диффузии с применением приближенного коэффициента вихревой диффузии. Для расчета уравнения сохранения массы при дисперсии возле линейного источника на уровне и выше уровня земли используется метод момента концентрации, который может использоваться при расчетах на ЭВМ. Уравнения моментов решаются численными методами для нулевого, второго, третьего и четвертого моментов продольного распределения концентрации.

Результаты данной работы согласуются с имеющимися аналитическими решениями для нейтральных течений. Статистические свойства диспергирующего облака используются для нахождения постоянных, которые появляются в лагранжевой гипотезе подобия Бетчелора для широкого диапазона условий устойчивости.